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Reasoning by CAS is a dead end !

Why is reasoning by CAS a dead end, particularly in education ? Because there is specific technology built for reasoning, which presently is entering practice of mathematics and of engineering.

These are the crucial differences in building educational tools in top of either CAS or TPS (we exemplary use Mathematica for CAS and Isabelle for TPS):

(1) Type system: In Isabelle's 'lemma "1 + 2 = (3::int)'" a click on "+" leads to "Groups" with algebraic laws, and a click on "int" leads to the wellknown equivalence relation deducing integers from naturals. In Mathematica such kinds of clicks are not enabled, they would lead to C++ code.

(2) Compositionality: CAS cope with mathematics stuff which cannot be done reliably -- in the hands of experts this is appropriate, but integration into larger software systems causes security risks. On the other side, in TPS all knowledge is proven and algorithms can be verified.

(3) Knowledge representation: In Mathematica knowledge is represented as C++ code. In Isabelle mathematics knowledge is represented in a human readable format, as shown already in (1).

(4) Scope of modeling mathematics: CAS covers only computation in mathematical problem solving. So students frequently feel overwhelmed by CAS.

On the other hand, TPS provide powerful concepts and tools for all phases of solving mathematical problems: in the modeling phase the type system makes code for checking user-input superfluous; in the specification phase predicates specifying problem-types and algorithms help to check choices for those; in the solving phase theorems are available (pt.(3)) for creating (and justifying!) steps in a calculation; etc.

Conclusion:

CAS are clumsy tools for educational purposes and for integration into educational tools. TPS are built for reasoning, and since reasoning is the fundament of mathematics, software built upon TP-components is promising for the future of mathematics education.